MULTIPLE DESCRIPTION IMAGE CODERS USING WHITENING TRANSFORMS: ANALYSIS AND COMPARATIVE STUDY

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ABSTRACT

A general two-stage multiple description coding (MDC) scheme using whitening transforms is analyzed. It represents the original image in a form of a coarse image approximation and a residual image. The coarse approximation is subsequently duplicated and combined with the residual image further split into two descriptions using a chessboard block transform coefficients rearrangement. We identify the importance of a good coarse approximation and explore different approaches for changing its resolution and coding it. The coder scheme is quite simple and yet achieves high performance comparable with other MDC methods.

1. INTRODUCTION

In the recent years, multiple description coding (MDC) has taken considerable attention as a method of communication over unreliable channels. MDC is a source-channel coding of information, which can be represented with different levels of quality. The source is encoded into several bitstreams (i.e. *multiple descriptions*) to be transmitted via *independent* channels. In the receiver, the source can be reconstructed by any single bitstream at lower but still acceptable quality. Higher quality is achieved by more bitstreams combined and the highest quality is achieved by all bitstreams received with no errors. By representing the source with different levels of quality MDC is similar to the layered coding. However, while the latter requires a correct reception of the base layer in order the enhancement layers to be useful, the former can reconstruct the source from any subset of bitstreams [1]. In order to achieve good source reconstruction from any description, all descriptions should have equal information content, i.e. they have to be similar to each other as they are similar and as close as possible to the source for the given bit budget. This makes the redundancy allocation an inherent issue of MDC. Under the assumption that some descriptions might be lost, reconstruction quality versus redundancy is the sought compromise in every MDC method. On the other hand, it is quite difficult to split good description of the source into the parts, which are independently useful [2], [1].

One of the first practical MDC methods, called multiple description scalar quantization (MDSQ), was proposed by Vaishampayan in [3]. In this method, the source variable to be transmitted is quantized by two coarse quantizers with overlapping quantization cells. Together, those quantizers produce fine quantization with smaller quantization cells.

Another MDC method is based on transform-domain processing [4], [5]. After a decorrelating transform, the uncorrelated coefficients are combined into pairs depending on their variances and then undergo pairwise correlating transform (PCT) yielding two descriptions, where the coefficients in one description are uncorrelated between each other, but correlated with the coefficients in the other description. If one description is lost, the correlation introduced in a known manner allows estimating the lost coefficient from the received one in the pair. The method shows quite good performance in the low-redundancy region, yet it has certain drawbacks. First, the variances of the initial coefficients are to be sent to the decoder, as they are used in the estimation procedure when one description is lost. Another problem is that the same coefficients in different blocks may have different variances. Thus, the blocks are to be classified into several classes based on their frequency properties. Nevertheless, there is often a mismatch between the real coefficient variance and one determined for the particular class of blocks. This mismatch is a source of additional distortions.

The above-mentioned problems have been addressed in [6] in the light of whitening the transform coefficients prior to PCT. Whitened coefficients have close variances that can be considered as equal. Correspondingly, there is no need to estimate and transmit those variances as they cancel in the estimator formula. The whitening transform is approximated by a subtraction of a down-sampled and coarsely coded image from the original image [6]. Then, a PCT as in [7] is applied to the coefficients of the residual (whitened) image. The resulting two descriptions of the residual image are combined with the duplicated version of the coarse image (shaper).

In this contribution, we adopt the whitening transform general scheme. We suggest modifications in the coarse approximation stage and in the residual image stage aimed at improving the quality for a given bit budget by a better redundancy management.

The paper is organized as follows. In Section 2, the general coder scheme is described. Next two sections present details about each of two stages: Section 3 deals with modifications in the coarse approximation coding stage while Section 4 deals with modifications in the residual image coding stage. Section 5 presents the numerical results and comparisons with other MDC methods, and Section 6 concludes the paper.

2. GENERAL CODER STRUCTURE

The general scheme of the method suggested in [6] is shown in Figure 1. The initial image is downsampled by two and then JPEG coded. Its decoded and interpolated version is subtracted from the initial image to approximate a whitening transform. DCT is applied to the residual image to get uncorrelated coefficients with approximately equal variance. They undergo PCT outputting two bitstreams. The JPEG coded coarse approximation is called *shaper* and is included into both descriptions. The redundancy in this method is mostly determined by duplicating the shaper but also extra redundancy is introduced by PCT. The method has given better results than the method in [4].

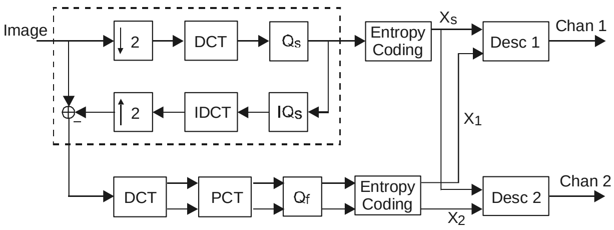

Figure 1. General scheme of method proposed in [6].

We modify the above-described method as shown in Figure 2. In our scheme the shaper (blocks bordered by the dashed line) is generated by decimation with an arbitrary down-scaling factor of *M* followed by a JPEG coder. We pay special attention to the way the image is decimated and interpolated. We favor a B-spline-based least square image resizing (biorthogonal projection) as it ensures a minimum loss of information [11]. Thus, most of image information is concentrated in the decimated image to be included in both descriptions. For the decimated image, a DCT-based coder is a reasonable choice. Alternatively, the shaper can be generated by a wavelet-based coder, e.g. SPIHT. In this case, the biorthogonal projection is inherently included in the scheme.

In our modification, the residual image is coded by a JPEG–like coder using a block transform (denoted by *T*). It can be either DCT or lapped orthogonal transform (LOT). The transform coefficients are finely quantized by a uniform quantization step (Q_r). Then, transform blocks are directly split into two parts in a *chessboard* manner and entropy–coded. One part together with the shaper form *Description* 1, while the second part combined again with the shaper form *Description* 2. Thus, each description consists of the coarse image approximation and *half* of the transform blocks of the residual image. Therefore, no extra redundancy is added in the residual image coding while generating two descriptions instead of one.

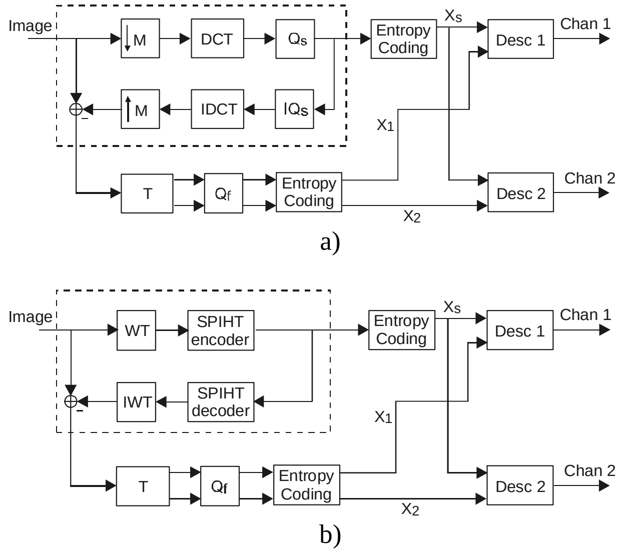

Figure 2. Varieties of proposed scheme: a) shaper is obtained by spline resizing and JPEG coding; b) shaper is obtained by SPIHT coding.

The obtained coder provides balanced descriptions both in terms of PSNR and bit rate. The amount of redundancy is also easily adjustable. The following two sections explain in details each stage of the coder. Also we give reasoning to use one or another method for each particular stage.

3. COARSE IMAGE APPROXIMATION

The idea of this stage is to concentrate as much information as possible into the shaper within strict bit rate constraints. We would also like to reduce the artifacts and distortions appearing in the reconstructed coarse approximation. To realize this idea we explore two alternatives: 1) Least squares image resizing prior to JPEG coding; and 2) Wavelet-domain SPIHT coding.

3.1. Least squares spline-based resizing and JPEG coding

A JPEG coder with a limited bit budget would use a large quantization factor applied directly to the original image thus causing unacceptable blocking artifacts. A better alternative, especially for low bit-rate coding, is to decimate the image first and to apply JPEG with more moderate quantization factor. The original image resolution is reconstructed by interpolation as a post-processing step. It has been proven by an analytical model and numerical analysis that by this approach the bit budget is kept the same while the visual quality and PSNR are higher [8]. The method in [6] also makes use of this approach as follows: the decimation is achieved by averaging over four neighbor pixels and the original resolution is reconstructed by nearest neighbor interpolation. This interpolation introduces blocking artifacts in the coarse approximation and as a result the residual image gets blocking artifacts as well.

In an attempt to concentrate more information in the coarse approximation and correspondingly to make the residual signal closer to white noise, we identify the need of a better interpolation and decimation method. Spline-based interpolation methods have shown their superiority in terms of quality and computational complexity [9], [10]. In the spline formalism, a continuous image model is fit over the discrete pixels, involving B-spline or other optimized piecewise-polynomial basis functions. It allows resampling the initial image at any arbitrary finer grid. As far as the image decimation is concerned, it has to be performed using functions being biorthogonal to the chosen interpolation function. This is the biorthogonal projection or least squares paradigm, which ensures image decimation with a minimum loss of information [11], [10]. Our practical implementation makes use of a near least squares method for image decimation proved to be effective for a wide range of decimation ratios [12].

The redundancy in our coder is only determined by the size (quality) of the shaper. Generally, there are two factors controlling the size of shaper (and hence, the redundancy). The first one is scaling (or interpolation) factor and the second one is the JPEG quantization factor. Using larger downsampling and quantization factors one can get lower level of redundancy, hence, lower quality of side reconstruction (reconstruction from only one description). Alternatively, using smaller downsampling and quantization factors, one can obtain higher quality side reconstruction. The quality of the two-channel reconstruction is determined mostly by quantization step used for quantization of LOT coefficients in the residual image.

3.2. Wavelet-based coding

An alternative to JPEG coding in obtaining good low bit-rate image approximation is some wavelet-based coding scheme. In general, wavelets provide smooth reconstruction of compressed images even for low bit rates. As they are functions for multiresolution analysis, there is no need of a preliminary decimation step. In fact, the wavelet decomposition is precisely an orthogonal or biorthogonal projection into the space of synthesizing (reconstruction) wavelet functions. Moreover, the best wavelets for compression have been generated via splines, e.g. the famous 9/7 synthesis/analysis wavelet pair. In our scheme we have involved the SPIHT coding and quantization algorithm [17].

4. RESIDUAL IMAGE CODING

In the original scheme (Figure 1) the residual signal is transformed into DCT domain and then an orthogonal PCT is applied to the DCT coefficients [6]. The idea is to catch some dependencies between pixels by DCT and then to pack the uncorrelated coefficients into correlated pairs to be sent as two descriptions. Therefore, this scheme adds additional redundancy to the one introduced by duplicating the shaper.

Conversely, our approach relies on a quality versus bit budget compromise achieved into the coarse approximation brunch. We speculate that our coarse approximation is as good as possible for the given bit budget and the residual image, therefore, should be closer to white noise. Thus, one can say that residual signal is less informative, and there is no need to introduce redundancy to this signal. Respectively, the total redundancy is added by only duplicating the base layer (shaper). We essentially aim at avoiding redundancy in the residual image coding.

The residual image coding in our method is done by a block transform, e.g. blocks of 8×8 pixels are considered. If all coefficients are sent by one description only, there is no redundancy, provided a proper block transform has been chosen. To generate MDC, the blocks are simply split into two descriptions in a *chessboard* manner.

We explore LOT and DCT as block transforms well suited for the residual image coding.

4.1. Coding of the residual signal with block DCT

The residual image is transformed using 8×8 DCT. Then, all transformed blocks are finely quantized with a scalar quantizer using a constant quantization step Q_f. The transform blocks are split between two descriptions in a chessboard manner and entropy coded separately.

4.2. Coding of the residual signal with lapped orthogonal transforms (LOT)

LOT is an alternative to DCT when the quality of the shaper is not good enough. In such cases some blocking artifacts can be encountered if the image reconstruction is based on one description only. LOT can efficiently smooth block borders based on the overlapping windows it uses.

By LOT, each signal block of size N is mapped into a set of N basis functions, each of them is longer than N samples, i.e. overlapping over adjacent blocks [13].

Given x is the original input vector of length MN, vector y of transformed coefficients of all M blocks is given by

$$y = T'x,$$

where T' is the transpose of T, which is given by

$$T = \begin{bmatrix} P_1 & & & & 0 \\ & P_0 & & & \\ & & \ddots & & \\ & & & P_0 & \\ 0 & & & & P_2 \end{bmatrix},$$

where P_0 is a $L \times N$ matrix that contains the LOT basis for each block, and P_1 and P_2 are the LOT matrices for the first and last blocks that should be slightly different. $L = 2N$.

The orthogonality of T is ensured by two conditions. First, the columns of P_0 have also to be orthogonal, i.e.

$$P_0' P_0 = I ,$$

where I is the identity matrix. Second, the overlapping functions of neighboring blocks have to be orthogonal as well,

$$P_0' W P_0 = P_0' W' P_0 = 0 ,$$

where the shift operator W is defined by

$$W = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix},$$

with the identity matrix above of size N [13].

For the 2D case the LOT's are implemented in a separable manner.

In our coder we use Malvar's LOT [13]. The overlapped blocks of the size 16×16 in a spatial domain correspond to 8×8 blocks in the transform domain. Next steps, i.e. quantization by a uniform quantization step Q_r and chessboard-like splitting into two parts are essentially the same like in the case of DCT block coding.

4.3. Reconstruction when one description is lost

When the decoder receives both descriptions, the reconstruction is straightforward. In case of one-channel reconstruction, the lost coefficients are just filled with zeros. Then, the inverse quantization and inverse transform are applied. The shaper can be obtained from the received description and added to the reconstructed residual image.

It is quite clear that this kind of reconstruction is appropriate when using DCT for coding of the residual image. It was also found that it is the most appropriate way of the reconstruction when using LOT for the residual image coding.

In [14] and [15] it was shown that when reconstructing the original image from only one description, setting the lost coefficients equal to zero produces severe artifacts. Thus, [14] and [15] present methods for estimation of the lost coefficients. In [14], the lost LOT coefficients are estimated as the mean of corresponding coefficients in the neighboring blocks. In [15], it was proposed to use iterative procedure using maximally smooth recovery method. Moreover, the family of LOT transforms with advanced reconstruction capabilities was presented in [14]. However, it was found that for coding of the residual zero-mean signal these methods work worse than just filling the lost coefficients with zeros. This fact is probably connected with high frequency nature of the residual signal that does not allow the estimation of the lost LOT block from the neighboring blocks.

The numerical results together with figures are presented in the next sections. The next section also compares the results obtained with our coder with the results obtained by [6].

5. SIMULATION RESULTS

In this section, varieties of our method are explored and compared between themselves and with the MDC algorithms using whitening transform [6]. For the evaluation, we applied our method to the test image Lena (512×512, 8 bpp). For each experiment we generated two rate-distortion curves. First one shows the reconstruction PSNR versus bitrate under the assumption that all (i.e. two) descriptions are received. The second one illustrates the case, when one description is lost. It is obtained by taking the mean result of two descriptions used separately to reconstruct the image.

5.1. Proper decimation and interpolation

In the first experiment, we compare different decimation and interpolation methods to produce the shaper. As for the residual image coding, we fix it to perform block transform coding, involving Malvar's LOT [13]. We apply three decimation/interpolation methods. The first is based on decimation by 2 by averaging over four nearest points and nearest neighbor interpolation, similarly to [6]. Second is DCT-based decimation and interpolation [18], and the third is a near least squares B-spline-based decimation and interpolation [12]. Those three approaches have been combined with the JPEG coder to get the coded shaper. Additionally, the shaper was obtained by a wavelet-domain SPIHT coding. Figure 3 shows the reconstruction results when both descriptions have been received and Figure 4 shows the results when one description is received only. As can be seen among JPEG methods proper anti-aliasing decimation and interpolation give substantial improvement. There, splines and DCT are quite competitive as pre- and post- processing functions however the spline-based method is computationally less costly. Among all methods, wavelet-based SPIHT gives superior results.

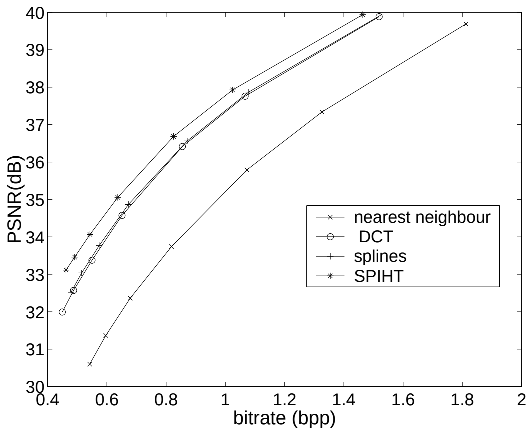

Figure 3. Central PSNR of overall scheme using different interpolation methods.

In our experiments we have used linear splines for interpolation and their biorthogonal counterparts for decimation. Higher order splines would give better results in a pure decimation/interpolation setting. However, the JPEG quantization generates artifacts and the subsequent higher-order interpolation makes them better visible. Linear interpolation plays an additional smoothing effect to these artifacts. What is more important is the least squares setting where the image is properly decimated subject to the chosen interpolation method.

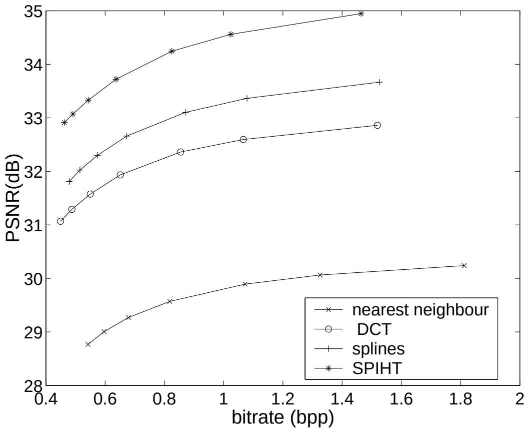

Figure 4. Mean side PSNR of overall scheme using different interpolation methods.

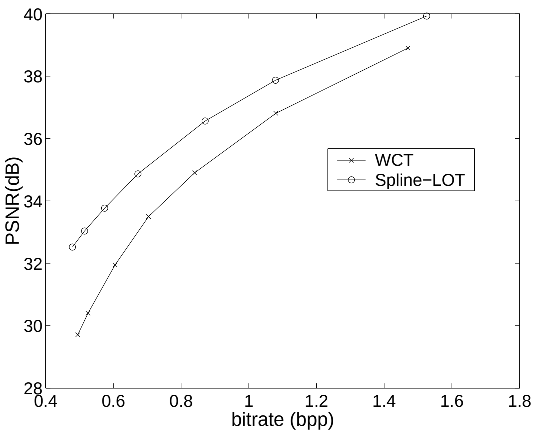

Figure 5. Comparison of Spline-LOT and WCT coder if both descriptions are received (central PSNR). Q_s=0.7.

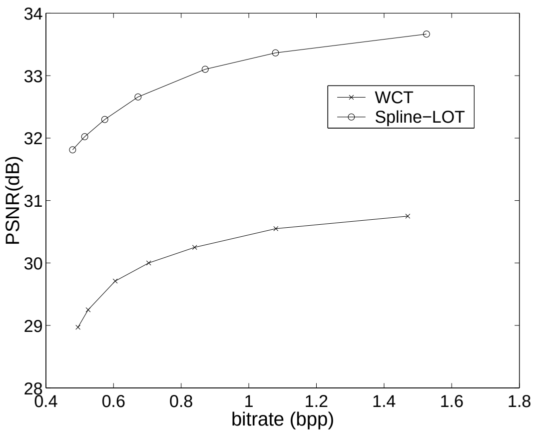

Figure 6. Comparison of Spline-LOT and WCT coders if one description is received (mean side PSNR). Q_s=0.7.

Figure 5 and Figure 6 compare Spline-LOT method with the method introduced in [6] (denoted by WCT). Spline-LOT coder sufficiently outperforms WCT coder both for central and side reconstruction mainly due to the adequate decimation/interpolation.

5.2. Shaper coding and quantization

Next, we explore how the shaper quality works on the total reconstruction quality. Again, our residual image coder is a LOT-based one, while the shaper coder is based on least squares spline decimation/interpolation and JPEG with different quantization factor (denoted as Spline-LOT). The shaper quantization factor Q_s is determined as a multiplication factor applied to DCT coefficients before their quantization. Figure 7 shows the results for central reconstruction (from two descriptions) and Figure 8 shows the one-description reconstruction results.

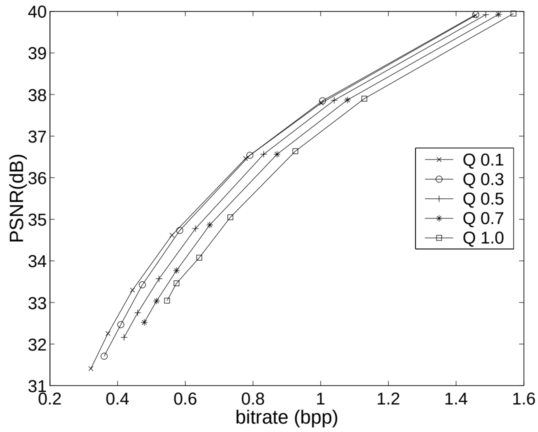

Figure 7. Rate-distortion performance of Spline-LOT coder for different values of Q_s. Central PSNR.

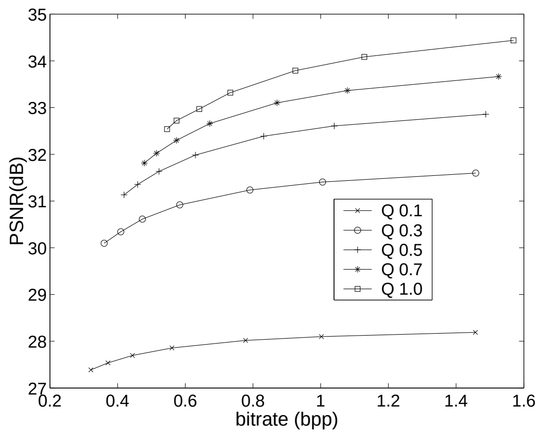

Figure 8. Rate-distortion performance of Spline-LOT coder for different values of Q_s. Mean-side PSNR.

One can see a higher quantization factor slightly reduces the PSNR for central reconstruction but at the same time increases the PSNR when one description is lost. By a finer quantization we thereby provide more bit rate to the shaper. Thus, we introduce more redundancy that improves the side reconstruction.

In addition, the rate-distortion curves for central reconstruction have much steeper slope than rate-distortion curves for side channel reconstruction. It evidences that finer quantization of residual image results in better central reconstruction but has little influence on side channel reconstruction.

The next algorithm uses SPIHT [17] for coding the shaper signal (denoted SPIHT-LOT). To produce results comparable with Spline-LOT, the bit-rate for shaper was chosen in a way to produce the shaper (base layer signal) of approximately same size. The results are parameterized by the shaper bit rate (sbr) and are shown in Figure 9 and Figure 10. It can be seen that PSNR for central and side reconstruction are better than these for Spline-LOT due to the superiority of SPIHT algorithm over the JPEG scheme for low bit rates.

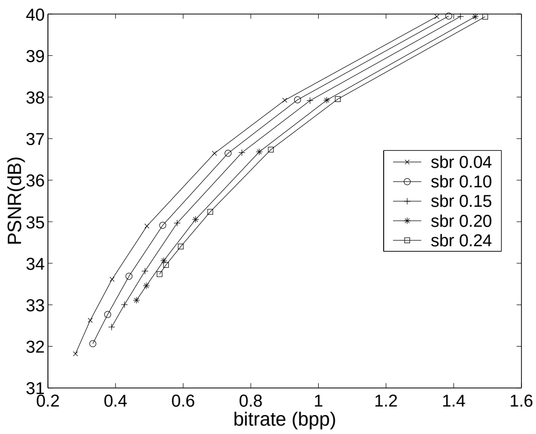

Figure 9. Rate-distortion performance (central PSNR) of SPIHT-LOT coder for different shaper bit rates.

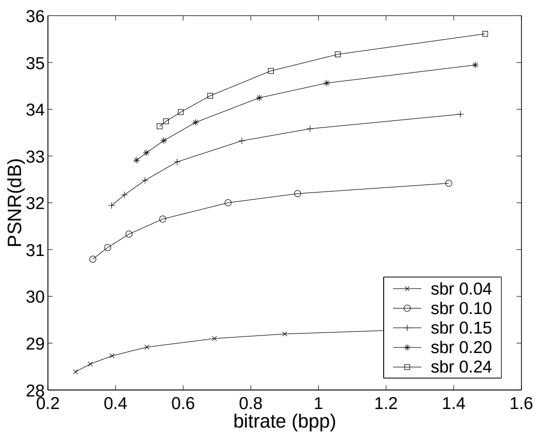

Figure 10. Rate-distortion performance (mean-side PSNR) of SPIHT-LOT coder for different shaper bit rates.

5.3. Residual signal coding

5.3.1. *LOT versus PCT*

In the next experiment, we fix the shaper coding to spline resizing and JPEG coding. We change the residual signal coding schemes, as follows: first, we apply the method from [6] involving DCT and subsequent PCT (denoted Spline-PCT), second we apply Spline-LOT. The results of those two approaches are quite close, as seen in Figure 11 and Figure 12.

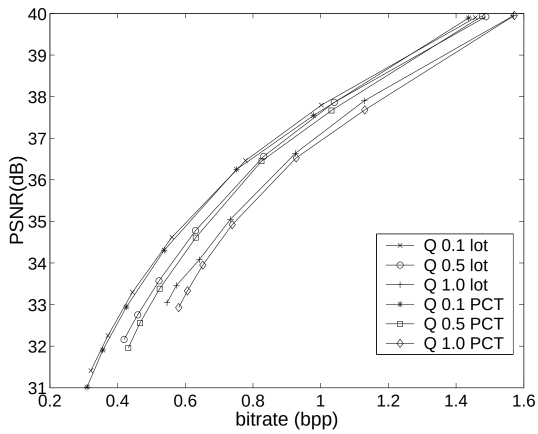

Figure 11. Comparison of Spline-LOT and Spline-PCT coders if both descriptions are received (central PSNR) for different values of Q_s.

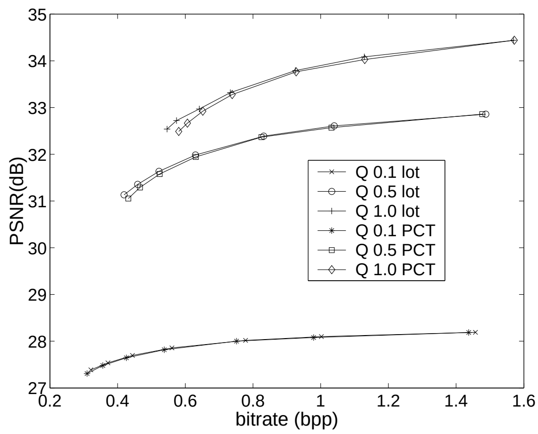

Figure 12. Comparison of Spline-LOT and Spline-PCT coders if one description is received (mean side PSNR) for different values of Q_s.

5.3.2. *DCT versus LOT*

We replace the LOT with DCT as the block transform and keep the same chessboard-like way of generating two descriptions (denoted Spline-Chess). Figure 13 and Figure 14 show the rate-distortion functions for Spline-Chess for different quantization factors.

A direct comparison between Spline-LOT and Spline-Chess is shown in Figure 15 and Figure 16. Quite surprisingly, in terms of PSNR, the Spline-Chess coder is competitive and even better than expected to be superior, Spline-LOT coder. While the latter is showing less blocking artifacts, it is not as efficient as DCT in compressing the high-frequency residual image. Originally, LOTs have been optimized to compress low frequency signals [13]. One can speculate that using transforms which are optimized for higher frequency content images could give certain improvement in the presented scheme.

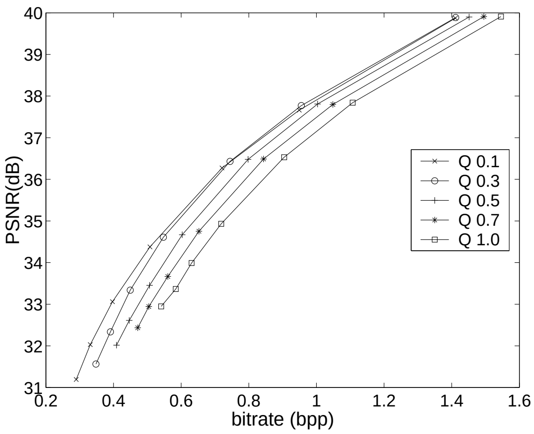

Figure 13. Rate-distortion performance (central PSNR) of Spline-Chess coder for different values of Q_s.

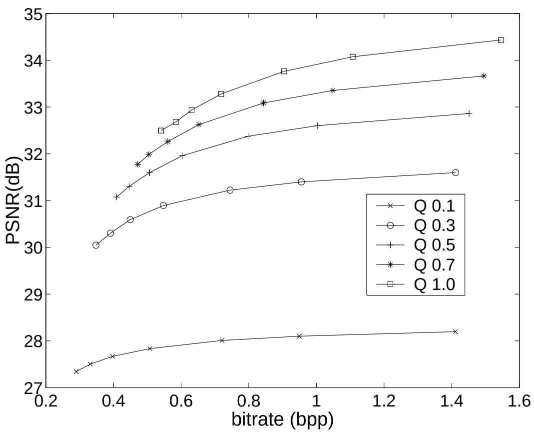

Figure 14. Rate-distortion performance (mean-side PSNR) of Spline-Chess coder for different values of Q_s.

The experiments with DCT in the residual image coding emphasize once again the importance of a good shaper coding. If the quality of the shaper is low, the blocking (chessboard-like) artifacts are more visible caused by reconstruction of neighboring blocks with different quality. However, if the shaper quality is good enough, then, for most of the images those kinds of artifacts are not visible. At least, they do not look visually more annoying than the artifacts caused by the coding of the residual image by LOT or PCT. Moreover, using DCT for coding of the residual image yields a slightly smaller bit rate comparing to LOT and PCT.

6. CONCLUSIONS

We have developed a practical MDC method that improves the two-stage scheme proposed previously in [6]. However, our premises for coder optimization were different from ones in [6]. The first stage of our coder employs spline interpolation to obtain the image with lower resolution, which is then coded and sent to both channels. This coarse image is coded in a way to have a lower bit rate, yet being smooth and providing satisfactory quality. Then, properly interpolated, this image is subtracted from the original one, yielding a residual (detail) image. We spend no redundancy in coding two descriptions out of it. To achieve this, a chessboard splitting of block transform coefficients is applied.

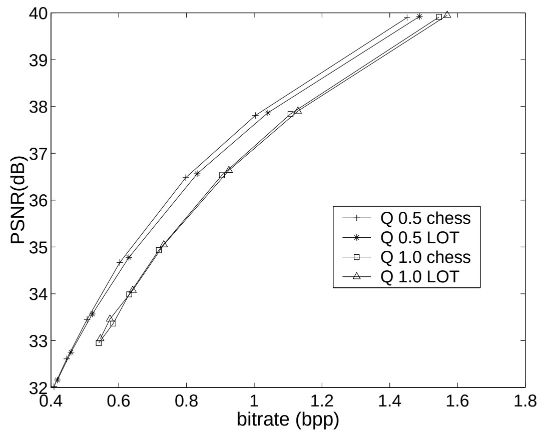

Figure 15. Comparison of Spline-Chess and Spline-LOT coders if both descriptions are received (central PSNR) for different values of Q_s.

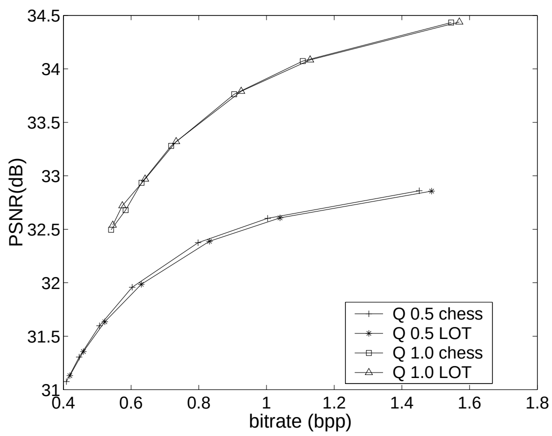

Figure 16. Comparison of Spline-Chess and Spline-LOT coders if one description is received (mean side PSNR) for different values of Q_s.

Two block-transform coders were compared for coding of the residual image. The simpler DCT-based coder showed competitive results to the LOT-based one. While the latter was expected to yield reconstructed images with less blocking artifacts, the good results for the former prove that we have achieved a residual image as high-frequency (noisy-like) as possible and correspondingly better compressible by DCT. The improved whitening effect is due to the adequate decimation/interpolation scheme we have applied based on biorthogonal projection (either spline or wavelet).

Our MDC method showed better performance comparing to the method in [6] both for reconstruction from one and two descriptions.

The further development of this coder may employ using suitable wavelet transforms for coding the residual signal. An application of this method for video coding is also to be considered.

7. REFERENCES

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